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REDUCTION OF ASTROGEODETIC DETERMINATIONS ON THE UNIQUE SYSTEM

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Abstract. Determinations in geodetic astronomy, namely, determination of time, longitude, latitude and azimuth from star observations, have been made during a long period by different persons. Positions of observed stars have been taken from different catalogues and in diferent reference systems. In order that results of such determinations will be mutually comparable, it is necessary to make the reduction of stellar positions on the unique reference system. Consequently, results of astrogeodetic determinations will also be reduced to the same system. Starting from the fact that the position of a point is determined only related to something, it is possible to calculate systematic differences of particular catalogues and catalogues used for the materialisation of the chosen reference system. Obtained stellar positions -, and proper movements - systematic errors, enable the changement from standard epoch on another one (the moment of observation), so that stellar positions are reduced to a single system. The proposed model is checked at three clssical methods of geodetic astronomy and applied to results of general Stevan Bošković's astrogeodetic determinations, made in the first decade of the XX century.

1. INTRODUCTION

Measurements in geodetic astronomy, or more precisely, determinations of astronomical time, plumb-line direction (natural coordinates) and of astronomical azimuth of geodetic sides are performed with high-precision instruments also enabling to obtain high-precision results. However, the results of determinations done nowadays indicate a disagreement between precision and accuracy (internal and external accuracy). This disagreement is due, among others, also to the importance of systematic and random errors in the positions of stars used in these determinations.

In order to answer concerning the cause of the disagreement it is necessary to analyse the methods of determination used in geodetic astronomy from the point of view of the presence of star-position errors in the determination results because the same position errors have a different influence on the determination results depending on the applied method and the star position in the horizontal system. On the other hand it is necessary to estimate the systematic and random errors in the positions of stars used in the determinations.

The determinations of geodetic astronomy have been carried out over a sufficiently long time interval and, consequently, any successful use of star coordinates needs their positions to be reduced to a unified system. In the present case the system of the FK5 Catalogue is of interest. For this reason the subject is the relationship between FK5 and other catalogues.

The main objective of the present work is, on the one hand, to assume a model for reducing star positions from different, already used, catalogues to a unified system, in particular to a system where FK5 serves as its reference frame. On the other hand, the objective of the study is to estimate the star positions reduced to this system, no matter whether they are from FK5, itself, or from another catalogue. In this way a reliable frame of studies for necessities of geodesy, geophysics or geodynamics would be obtained.

In view of the already existing results a hypothesis can be formulated that the errors of star positions given in earlier catalogues are important to the existence of the disagreement between the precision and accuracy for the determinations carried out in earlier epochs where star positions from these catalogues were used. With regard to the high precision of modern instruments used in geodetic astronomy there are also reasons to test the hypothesis concerning the importance of the influence of errors in star positions on the modern determinations of geodetic astronomy.

The present work is based on the assumption that a sufficiently large body of data (results) exists in fundamental and ephemeride astronomy concerning the properties of relevant catalogues, the values and accuracy of astronomical constants and, above all, the accuracy of proper motions so that a reliable estimation of the accuracy of star positions for epochs of interest can be done.

In "Studija o stanju dosadašnjih radova na astrogeodetskoj mreži SFRJ" (Study about the Works on Astrogeodetic Network of the SFRY done up to now) there are tasks solved through astrogeodetic measurements where these measurements are used either independently or combined with gravimetric or satellite ones. An inspection of the works carried out over the territory of Yugoslavia is characterised by a statement that the determinations undertaken before the First World War could be of interest only with the results of latitude determination. Generally, the accuracy of latitude determination, also of azimuth one, is low due to the technological level of that time. This concerns the determinations performed by Stevan P. Bošković between 1900 and 1911. The results were published as "Skretanja vertikala u Srbiji" (Plumb-Line Deflections in Serbia, Bošković 1952) in an edition of the Serbian Royal Academy of Sciences.

It is clear that in the joining of the results of Bošković's determinations the ones of modern astrogeodesy their insufficient accuracy in view of the necessities and facilities of the present epoch should be taken into account. However, if established how large the presence of the errors in star coordinates in the total budget of astrogeodetic determinations is, this part could be removed. In other words, by reducing the star positions to some of the modern systems such as, for instance, FK5 the accuracy is enlarged just in the part which is dependent of the coordiantes of the observed stars.

2. POSITION ERRORS

In a catalogue the data for every star concerning its position, proper motion, number of observations and mean epoch are usually followed by the ones concerning the mean errors in the position and proper motion. These individual mean errors, together with the mean errors of the catalogue system, offer the insight into the accuracy of star positions. (As said above, the term "catalogue system" indicates the comprehensiveness of the data defining star positions also loaded by systematic errors. In view of this the notions of catalogue system and reference system should be distinguished.) Since the system of a catalogue is obtained from numerous star observations during a sufficiently long time interval, it is usual to present the mean errors of catalogue system in special tables through the root-mean-square (rms) errors for the systems of right ascensions, declinations and proper motions, followed by the data on the mean epoch of observations.

The systematic errors are calculated by using stars common to given catalogues and are expressed through the differences of their positions averaged in zones. These differences can be applied for the purpose of changing the catalogue system, i. e. of reducing to the system of a selected catalogue. For instance, by using the systematic differences (FK5 - FK4), (FK4 - FK3) and (FK3 - NFK) the (FK5 - NFK) ones can be calculated in an indirect way. These differences should be understood as corrections to the NFK Catalogue and this is the reducing of the positions of NFK stars to the FK5 system.

The determinations of plumb-line deflections in Serbia performed in the beginning of the XX century by S. P. Bošković are of interest here. Bošković's observing programme contained bright stars only belonging to the Group of so-called Auwers' stars. These stars are in the NFK Catalogue and they are designated as 1 to 925. Due to this in what follows the consideration of random and systematic errors for the positions of these stars will be given.

2.1. RANDOM ERRORS OF POSITIONS OF FUNDAMENTAL STARS

Compared to an ideal system the mean error of a star position will contain the part representing the mean error of the catalogue fundamental system, itself, and the part representing the individual mean position error in the given system (Mueller, 1969)

$$E^2 = m^2 + M^2 \ . \tag{1}$$

Here, the mean error of the system m for epoch T is calculated by using the following formulae

$$(m_\alpha \cos\delta)_T{}^2 = (m_\alpha \cos\delta)_{T_0}{}^2 + \left(\frac{T - T_{s\alpha}}{100}\right)^2 (m_\mu \cos\delta)^2 \ , \tag{2}$$

$$(m_\delta)_T{}^2 = (m_\delta)_{T_0}{}^2 + \left(\frac{T - T_{s\delta}}{100}\right)^2 (m_{\mu'})^2 \ , \tag{3}$$

whereas the mean error of a star position in this system M for the same epoch according to the formulae

$$(M_\alpha \cos\delta)_T{}^2 = (M_\alpha \cos\delta)_{T_0}{}^2 + \left(\frac{T - T_{0\alpha}}{100}\right)^2 (M_\mu \cos\delta)^2 \ , \tag{4}$$

$$(M_\delta)_T{}^2 = (M_\delta)_{T_0}{}^2 + \left(\frac{T - T_{0\delta}}{100}\right)^2 (M_{\mu'})^2 \ . \tag{5}$$

The mean errors of the FK4 (Fricke and Kopff, 1963) and FK5 (Fricke et al., 1991) systems are given in the forewords of these catalogues. Since here the error of the system is numerically approximately equal to the average value of the individual error of the star position in the catalogue, it can be assumed that the same is valid for the NFK (Peters, 1907) and FK3 (Kopff, 1937) systems in view of how these systems were formed. As for NFK, the values are calculated bearing in mind the fact that the position errors for its stars are 1.5 times as large as those in FK3, whereas in the case of the proper motions the corresponding ratio is equal to 1.8. These data are added to the table presenting the mean errors of the coordinates and those of the centennial proper motions in FK3 (Zverev, 1950, p. 78).

The data from the catalogues FK3, FK4 and FK5 enable the calculating of the mean values of individual mean errors in the positions and proper motions of stars for each of the catalogues separately. In the present calcualtion Auwers' stars with declinations greater than -20 are taken because some of them were used by Bošković in his determinations.

Table 1. presents the results obtained by the present author. The catalogue designation is in the first column. The second and the third ones contain the mean values of the mean epochs of star observations $(T_0)_\alpha$ and $(T_0)_\delta$. The mean individual errors of right ascensions and declinations $M_\alpha \cos\delta$ and M_δ are in the fourth and sixth columns, respectively, whereas the corresponding errors of the proper motions over century $M_\mu \cos\delta$ and $M_{\mu'}$ are in the fifth and seventh columns, respectively. It should be mentioned that the errors in NFK are estimated on the basis of those in FK3.

Table 1. Mean values of individual mean errors in positions and proper motions of Auwers' stars with declinations greater than -20

Cat	$(T_0)_\alpha$	$(T_0)_\delta$	$M_\alpha \cos\delta$	$M_\mu \cos\delta/cy$	M_δ	$M_{\mu'}/cy$
NFK	1870	1870	$\pm0\overset{s}{.}0030$	$\pm0\overset{s}{.}0200$	$\pm0\overset{''}{.}050$	$\pm0\overset{''}{.}270$
FK3	1901	1897	$\pm0\overset{s}{.}0019$	$\pm0\overset{s}{.}0110$	$\pm0\overset{''}{.}031$	$\pm0\overset{''}{.}151$
FK4	1915	1911	$\pm0\overset{s}{.}0014$	$\pm0\overset{s}{.}0063$	$\pm0\overset{''}{.}024$	$\pm0\overset{''}{.}097$
FK5	1950	1939	$\pm0\overset{s}{.}0008$	$\pm0\overset{s}{.}0028$	$\pm0\overset{''}{.}017$	$\pm0\overset{''}{.}051$

The data from the catalogues and Tables 1. enable to obtain by applying formulae (1)-(5) the average mean errors in positions of the stars used in Bošković's determinations of plumb-line deflections in Serbia. The errors are calculated for the beginning of the XX century: just for the mean epoch of Bošković's works approximately equal to 1906.0.

The average mean errors in right ascension $E_\alpha \cos\delta$ and declination E_δ with respect to an ideal system calculated for the NFK, FK3, FK4 and FK5 catalogues are presented in Table 2.

Table 2. Average mean position errors for stars used in Bošković's astrogeodetic determinations - epoch 1906.0.

Cat	$E_\alpha \cos\delta$	E_δ
NFK	$\pm 0\overset{s}{.}015$	$\pm 0\overset{''}{.}20$
FK3	$\pm 0\overset{s}{.}004$	$\pm 0\overset{''}{.}06$
FK4	$\pm 0\overset{s}{.}004$	$\pm 0\overset{''}{.}03$
FK5	$\pm 0\overset{s}{.}003$	$\pm 0\overset{''}{.}03$

2.2. SYSTEMATIC ERRORS IN POSITIONS OF FUNDAMENTAL STARS

Since the random errors in the positions of the most important stars in the fundamental systems NFK, FK3, FK4 and FK5 are estimated, the next step is to determine the systematic errors of the fundamental catalogue NFK. If FK5 is taken as a reference catalogue, these errors will be as the systematic differences NFK-FK5. With regard that for the purposes of the present paper the reducing of Bošković's observations to the system of the catalogue FK5 is of interest, what is looked for is the corrections of the NFK system. In other words, the differences FK5-NFK in the corresponding parameters are calculated. This calculation is done in the following phases: 1) FK4 reduced to FK5; 2) FK3 reduced to FK4; 3) NFK reduced to FK3. Bearing in mind the mean epoch of Bošković's observations the reducing of NFK to FK5 takes place for the epoch T=1906.0.

1) As for the FK4 system, the first step is to correct the position of the vernal-equinox point following the formula

$$E(T) = +0\overset{s}{.}035 + 0\overset{s}{.}085\left(\frac{T-1950}{100}\right) , \tag{6}$$

to calculate the reducing of FK4 to the FK5 system (differences FK5- FK4). Here the FK4 equator remains unchanged.

The regional differences FK5-FK4, given in the form of tables on pages 87-90 of the FK5 Catalogue, are used for the purpose of finding the differences of right ascensions and declinations according to the classical formulae:

$$\begin{aligned} (\Delta\alpha\cos\delta)_{1950} &= [(\Delta\alpha_\alpha + \Delta\alpha_\delta + \Delta\alpha_m)\cos\delta]_{1950} \\ \Delta\mu\cos\delta &= (\Delta\mu_\alpha + \Delta\mu_\delta + \Delta\mu_m)\cos\delta \ ; \end{aligned} \tag{7}$$

$$\begin{aligned} (\Delta\delta)_{1950} &= [\Delta\delta_\alpha + \Delta\delta_\delta + \Delta\delta_m]_{1950} \\ \Delta\mu' &= (\Delta\mu'_\alpha + \Delta\mu'_\delta) \ . \end{aligned} \tag{8}$$

The reducing of the systematic differences to the epoch T is done without the factor cos

$$(\Delta\alpha)_T = (\Delta\alpha)_{1950} + \Delta\mu \left(\frac{T-1950}{100}\right) \quad (9)$$
$$(\Delta\delta)_T = (\Delta\delta)_{1950} + \Delta\mu' \left(\frac{T-1950}{100}\right) .$$

2) In the reducing of the FK3 system to that of FK4 one should bear in mind that the equator correction is done according to the formula

$$D(T) = -0\overset{''}{.}017 + 0\overset{''}{.}097\left(\frac{T-1928.4}{100}\right) , \quad (10)$$

but this correction is theoretically zero at the poles.

The regional errors for the FK3 Catalogue, i. e. the differences FK4-FK3 for the year 1950 are given on the pages 131-134 of the FK4 Catalogue. For the epoch 1906.0 the calculation is done according to formulae (7), (8) and (9), but it is taken into account that the values $\Delta\alpha$ and $\Delta\mu$ for the stars with declinations exceeding 60 in their modulus are reduced to the equator (multiplied by $\cos\delta$).

3) The corrections of the NFK system are given directly on pages 84 - 87 of the FK3 Catalogue in Table 3 which contains differences from 1800 to 1930. Though not explicitely specified in the tables, it can be concluded from the data-treatment procedure used for the purpose of improving the NFK system that the differences $\Delta\alpha_\delta$ for declinations between -60 and +60 are given directly, whereas for ones beyond this interval are reduced to the equator.

Table 3. Systematic differences $\Delta\alpha$ - reducing of NFK to FK5 system for epoch 1906.0

$\alpha \setminus \delta$	$+70^o$	$+50^o$	$+30^o$	$+10^o$	-10^o
0^h	$-0\overset{s}{.}196$	$-0\overset{s}{.}073$	$-0\overset{s}{.}060$	$-0\overset{s}{.}051$	$-0\overset{s}{.}043$
4^h	$-0\overset{s}{.}201$	$-0\overset{s}{.}073$	$-0\overset{s}{.}064$	$-0\overset{s}{.}057$	$-0\overset{s}{.}049$
8^h	$-0\overset{s}{.}225$	$-0\overset{s}{.}088$	$-0\overset{s}{.}080$	$-0\overset{s}{.}073$	$-0\overset{s}{.}064$
12^h	$-0\overset{s}{.}217$	$-0\overset{s}{.}084$	$-0\overset{s}{.}066$	$-0\overset{s}{.}062$	$-0\overset{s}{.}055$
16^h	$-0\overset{s}{.}217$	$-0\overset{s}{.}087$	$-0\overset{s}{.}070$	$-0\overset{s}{.}064$	$-0\overset{s}{.}059$
20^h	$-0\overset{s}{.}204$	$-0\overset{s}{.}074$	$-0\overset{s}{.}060$	$-0\overset{s}{.}051$	$-0\overset{s}{.}044$
$\Delta\alpha_\delta$	$-0\overset{s}{.}210$	$-0\overset{s}{.}080$	$-0\overset{s}{.}067$	$-0\overset{s}{.}060$	$-0\overset{s}{.}052$

Therefore, the data presented with the fundamental catalogues FK5, FK4 and FK3 are used for the purpose of composing Tables 3. and 4. in which the systematic differences of the positions FK5 - NFK for epoch 1906.0 in declination zones from -20 to +80.

In Tables 3. and 4. the differences $\Delta\alpha$ and $\Delta\delta$ are grouped over zones of declination and right ascension. In this way we have a more detailed insight into the systematic errors of NFK here compared to FK5. The middle of a 20-degree declination zone is designated in the first row of both tables, whereas the one of a four-hour right-ascension zone is in the first column. The last row in both tables contains the differences in declination averaged over all the six zones of right ascension.

Tables 3. and 4. give the reducing of NFK to the system of FK5 and together with Table 2. they will serve in the estimation of the influences of errors in star coordinates on the results of Bošković's astrogeodetic determinations.

Table 4. Systematic differences $\Delta\delta$ - reducing of NFK to FK5 system for epoch 1906.0

$\alpha \setminus \delta$	$+70^o$	$+50^o$	$+30^o$	$+10^o$	-10^o
0^h	$+0.''10$	$+0.''15$	$+0.''10$	$+0.''25$	$+0.''08$
4^h	$+0.''10$	$+0.''18$	$+0.''09$	$+0.''25$	$+0.''10$
8^h	$-0.''02$	$+0.''04$	$-0.''03$	$+0.''11$	$-0.''01$
12^h	$+0.''05$	$+0.''10$	$+0.''04$	$+0.''18$	$+0.''03$
16^h	$+0.''07$	$+0.''16$	$+0.''11$	$+0.''24$	$+0.''11$
20^h	$+0.''06$	$+0.''05$	$-0.''02$	$+0.''13$	$+0.''02$
$\Delta\delta_\delta$	$+0.''06$	$+0.''11$	$+0.''05$	$+0.''19$	$+0.''06$

2.3. BOŠKOVIĆ'S DETERMINATIONS 1900 - 1911.

In first years of the XX century geodetic determinations aimed at the exact topographic measuring of Serbia. On this occasion significant disagreements concerning the cartographic works of former Austria-Hungary, Romania, Bulgaria and Serbia were discovered, just at the touching point of these states, i. e. in the north-east of Serbia of that time where the Carpato-Balkanic arc intersects these regions detaching longitudinally the Panonian bassein from the Pontic one. On account of this it is very important to solve the problem about the cause of the disagreement. Another important question is how to orientate the triangulation of Serbia and onto which spheroid to project it. Due to the connection between the geodetic works of that time and the activity of the Military Geographic Institute in Vienna, through which a similar connection arose with the analogous activity in the lands of Central Europe, it was decided to realise the orientation according to the activity of the Vienna Institute and following them to project the triangulation onto the Besselian spheroid as was usual then for all geodetic activity in Central Europe.

S. P. Bošković in the late XIX century during his studies in Pulkovo intuitively understood that, probably, the deviation of the plumb line from its normal position with respect to the ideally curved surface of the terrestrial spheroid was the cause for the geodetic, and automatically the cartographic, mostly longitudinal, disagreement in the lands of the Panonic and Pontic baseins. This might be a consequence of a local atraction perturbation in the intensity of gravity force due to the structure of the Carpato-Balkanic mountain arc. For this reason he planned already then geodetic and astronomical activities in order to verify if his intuition was correct.

For the purpose of realising his idea S. P. Bošković prepared two universal instruments for geodetic and astronomical observations, twelve chronometers, aneroids and thermometers. The ephemerides for star pairs are calculated for the purpose of time determination by using Tsinger's method for the points of all geographic latitudes in Serbia of that time and also the ephemerides for star pairs for the purpose of latitude determination by using Pevtsov's method and the ephemerides for the Polar Star for the purpose of azimuth determination by using the classical method. Bošković's plan was to measure simultaneously horizontal and vertical triangulation angles and astronomical measurements aimed at determining time, latitude and azimuth choosing for this a number of points on our highest mountains, as well as a number of points in our river valleys thinking that this was the best way to examine and discover the local atraction influences on the perturbation of the normal direction of the intensity of Earth gravity and due to this the deviation of the plumb line.

The first determinations at I - northern point of the Paraćin basis and at the highest top of the mountain of Rtanj in 1900 had very good results. Since the expeditions concerning these activities at chosen highest points of our mountains require strong physical efforts, Bošković organised the programme of geodetic and astronomical activities in such a way to be performed during the first years and afterwards points in our river valleys would be treated. In 1901 there were expeditions and measurements on Midžor, Trem and Jastrebac; in 1902 Veliki Streser, Petrova Gora, Kopaonik, Jankov Kamen and Tornik; in 1903 Mali Povlen, Deli Jovan, Veliki Sumorovac, Crni Vrh, Bukulja and Cer. Afterwards, in 1905, at the churches in Niš, Zaječar and Negotin; in 1906 at Pirot, Tija-Bara (near the monument) and at II point (northern) of the Vranje basis in the village of Zlatokop; in 1907 at Hisar (Leskovac), at the churches in Trstenik and Čačak; in 1908 at I point of Loznica basis - Starača and at Ozerovac near the Markovac bridge over the Morava. Later on, in 1909 on Avala, at Podgorica and Kulič (at the junction of the Morava and the Danube); finally in 1911 at Osojna near Kladovo and in Pirot at Tija-Bara. There were no astronomical activities in 1904, when Bošković's occupations were measurements of four bases, and in 1910 when his occupations were measurements of the Drina valley way between Zvornik and Rača in cooperation with the Vienna Military Institute of Geography.

The urgent works on the calculation of the trigonometric triangulation and the ones commenced on the new topographic measurements for the scale of 1:25,000, also the wars in 1912 and 1913, the urgent geodetic works aimed at new topographic measurements of the liberated parts of the country, as well as the wars between 1914 and 1920, made impossible this enormous astronomical material to be treated. However, it, as well as the triangulation, was not lost. It was transported through Albania to the island of Corfu, then to Thessaloniki and finally back to our country - it came to Belgrade in 1919. At that time the Belgrade Military Institute of Geography, in addition to its own specialist staff, got also a new one, Russian topographers who helped very much in the treatment of the astronomical data.

In Table 5. a presentation of points, with the year of astrogeodetic determinations and coordinates, is given. The column of latitudes contains Bošković's values, as well as the last one with the azimuth values.

Table 5. Points at which the plumb-line deflection was determined in Serbia 1900 – 1911 (λ – longitude of place where observations were performed; φ – Bošković's values for latitude; A – Bošković's values for azimuth).

	Place of Observation	year	λ	φ	A
1.	Paraćin	1900.	$-1^h\,25^m\!.6$	$+43^\circ\,50'\,31''\!.57$	$277^\circ\,00'\,42''\!.36$
2.	Rtanj	1900.	$-1^h\,27^m\!.6$	$+43^\circ\,46'\,39''\!.98$	$250^\circ\,27'\,47''\!.48$
3.	Midžor	1901.	$-1^h\,30^m\!.8$	$+43^\circ\,23'\,50''\!.52$	$170^\circ\,55'\,05''\!.48$
4.	Trem	1901.	$-1^h\,28^m\!.7$	$+43^\circ\,11'\,10''\!.10$	$64^\circ\,28'\,37''\!.81$
5.	Jastrebac	1901.	$-1^h\,25^m\!.8$	$+43^\circ\,22'\,58''\!.37$	$44^\circ\,16'\,58''\!.68$
6.	Streser	1902.	$-1^h\,29^m\!.0$	$+42^\circ\,37'\,39''\!.02$	$15^\circ\,40'\,28''\!.03$
7.	Petrova Gora	1902.	$-1^h\,26^m\!.1$	$+42^\circ\,59'\,55''\!.41$	$63^\circ\,15'\,27''\!.57$
8.	Kopaonik	1902.	$-1^h\,23^m\!.3$	$+43^\circ\,16'\,06''\!.24$	$135^\circ\,46'\,21''\!.21$
9.	Jankov Kamen	1902.	$-1^h\,21^m\!.1$	$+43^\circ\,20'\,23''\!.12$	$139^\circ\,31'\,56''\!.68$
10.	Tornik	1902.	$-1^h\,18^m\!.7$	$+43^\circ\,39'\,10''\!.12$	$201^\circ\,16'\,11''\!.63$
11.	Mali Povlen	1903.	$-1^h\,18^m\!.9$	$+44^\circ\,07'\,53''\!.08$	$29^\circ\,29'\,07''\!.65$
12.	Deli Jovan	1903.	$-1^h\,28^m\!.9$	$+44^\circ\,13'\,39''\!.75$	$59^\circ\,29'\,26''\!.37$
13.	Veliki Sumorovac	1903.	$-1^h\,26^m\!.6$	$+44^\circ\,19'\,01''\!.08$	$184^\circ\,53'\,13''\!.17$
14.	Crni Vrh	1903.	$-1^h\,23^m\!.6$	$+43^\circ\,51'\,41''\!.31$	$199^\circ\,06'\,29''\!.75$
15.	Bukulja	1903.	$-1^h\,21^m\!.9$	$+44^\circ\,17'\,59''\!.29$	$188^\circ\,34'\,13''\!.41$
16.	Cer	1903.	$-1^h\,18^m\!.0$	$+44^\circ\,36'\,14''\!.77$	$105^\circ\,00'\,58''\!.66$
17.	Church in Niš	1905.	$-1^h\,27^m\!.6$	$+43^\circ\,18'\,55''\!.62$	$218^\circ\,14'\,31''\!.58$
18.	Church in Zaječar	1905.	$-1^h\,29^m\!.2$	$+43^\circ\,54'\,08''\!.28$	$65^\circ\,34'\,25''\!.86$
19.	Church in Negotin	1905.	$-1^h\,30^m\!.1$	$+44^\circ\,13'\,39''\!.12$	$179^\circ\,05'\,34''\!.41$
20.	Pirot	1906.	$-1^h\,30^m\!.4$	$+43^\circ\,09'\,36''\!.47$	$288^\circ\,30'\,27''\!.85$
21.	Zlatokop	1906.	$-1^h\,27^m\!.3$	$+42^\circ\,31'\,03''\!.04$	$122^\circ\,02'\,43''\!.35$
22.	Hisar	1907.	$-1^h\,27^m\!.8$	$+42^\circ\,59'\,12''\!.76$	$221^\circ\,31'\,47''\!.66$
23.	Church in Trstenik	1907.	$-1^h\,24^m\!.1$	$+43^\circ\,37'\,16''\!.44$	$190^\circ\,50'\,30''\!.32$
24.	Church in Čačak	1907.	$-1^h\,21^m\!.4$	$+43^\circ\,53'\,38''\!.70$	$99^\circ\,27'\,12''\!.55$
25.	Starača	1908.	$-1^h\,16^m\!.8$	$+44^\circ\,34'\,30''\!.59$	$232^\circ\,06'\,56''\!.65$
26.	Ozerovac	1908.	$-1^h\,24^m\!.4$	$+44^\circ\,14'\,08''\!.20$	$225^\circ\,02'\,57''\!.46$
27.	Avala	1909.	$-1^h\,22^m\!.0$	$+44^\circ\,41'\,22''\!.50$	$194^\circ\,19'\,50''\!.30$
28.	Kulič	1909.	$-1^h\,19^m\!.6$	$+44^\circ\,42'\,52''\!.08$	$272^\circ\,02'\,05''\!.21$
29.	Podgorica	1909.	$-1^h\,24^m\!.1$	$+44^\circ\,40'\,59''\!.45$	$114^\circ\,27'\,53''\!.41$
30.	Osojna	1911.	$-1^h\,30^m\!.4$	$+44^\circ\,35'\,18''\!.79$	$74^\circ\,31'\,36''\!.01$

3. THE REDUCING TO THE FK5 SYSTEM

Essentially, the best way of reducing the results of Bošković's astrogeodetic determinations to the FK5 system is to use his original values of apparent star coordinates (right ascensions and declinations) in order to avoid various incertainties and speculations. After enquiring at several places, among others at the Archives of the Serbian Academy of Sciences and Arts, it has been established that Bošković's entire reduction material, more precisely the reduction sheets with Tsinger's and Pevtsov's pairs and Polar-Star observations, is in the Library of the Military Institute of Geography.

Table 6. The Corrections of Bošković's Astrogeodetic Determinations ($\Delta\varphi_c$ - correction in latitude calculation;$\Delta\varphi_{fk5}$ and ΔA_{fk5} - reducing to FK5 system; $\Delta\varphi_{cio}$ and ΔA_{cio} - reducing to CIO pole; $[\xi]$ - corrected component of plumb-line deflection in north-south direction; $[\eta]$ - corrected component of plumb-line deflection in east-west direction).

	Place of Observation	$\Delta\varphi_c$	$\Delta\varphi_{fk5}$	$\Delta\varphi_{cio}$	$[\xi]$	ΔA_{fk5}	ΔA_{cio}	$[\eta]$
1.	Paraćin	$-0.''32$	$+0.''22$	$+0.''05$	$-00.''15$	$-0.''08$	$+0.''07$	$-14.''07$
2.	Rtanj	$+0.''16$	$+0.''30$	$+0.''06$	$+07.''02$	$-0.''49$	$+0.''05$	$-06.''83$
3.	Midžor	$+0.''05$	$+0.''16$	$-0.''12$	$+08.''49$	$-0.''15$	$+0.''17$	$-01.''78$
4.	Trem	$-0.''09$	$+0.''03$	$-0.''08$	$+10.''26$	$-0.''26$	$+0.''18$	$-08.''40$
5.	Jastrebac	$-0.''48$	$+0.''11$	$-0.''06$	$+00.''67$	$-0.''28$	$+0.''18$	$-08.''78$
6.	Streser	$+1.''32$	$+0.''15$	$-0.''16$	$+05.''81$	$+0.''12$	$-0.''20$	$-11.''60$
7.	Petrova Gora	$-0.''13$	$+0.''09$	$-0.''19$	$-00.''93$	$-0.''14$	$-0.''13$	$-06.''94$
8.	Kopaonik	$-0.''21$	$-0.''01$	$-0.''19$	$-02.''81$	$-0.''39$	$-0.''09$	$-02.''85$
9.	Jankov kamen	$+0.''23$	$+0.''17$	$-0.''19$	$+02.''41$	$-0.''47$	$-0.''07$	$+02.''50$
10.	Tornik	$+0.''07$	$+0.''02$	$-0.''18$	$-03.''19$	$-0.''60$	$-0.''01$	$-01.''16$
11.	Mali Povlen	$-0.''29$	$+0.''02$	$-0.''09$	$+02.''94$	$-0.''10$	$-0.''30$	$-07.''32$
12.	Deli Jovan	$+0.''45$	$+0.''04$	$-0.''14$	$+05.''75$	$-0.''08$	$-0.''26$	$+02.''53$
13.	Veliki Sumorovac	$-0.''10$	$+0.''05$	$-0.''15$	$+00.''70$	$-0.''11$	$-0.''24$	$-04.''15$
14.	Crni Vrh	$+0.''29$	$-0.''05$	$-0.''17$	$+02.''37$	$-0.''13$	$-0.''18$	$-06.''05$
15.	Bukulja	$+0.''78$	$-0.''13$	$-0.''18$	$+05.''37$	$-0.''12$	$-0.''17$	$-06.''04$
16.	Cer	$+1.''17$	$-0.''22$	$-0.''19$	$+04.''06$	$-0.''18$	$-0.''12$	$-07.''10$
17.	Church in Niš	$+0.''42$	$+0.''16$	$+0.''01$	$+02.''29$	$-0.''30$	$-0.''24$	$-11.''39$
18.	Church in Zaječar	$+0.''37$	$+0.''01$	$-0.''02$	$+00.''76$	$-0.''07$	$-0.''24$	$+14.''95$
19.	Church in Negotin	$+0.''08$	$+0.''42$	$-0.''05$	$+03.''75$	$+0.''04$	$-0.''23$	$+14.''60$
20.	Pirot	$+0.''09$	$+0.''08$	$+0.''07$	$-02.''06$	$-0.''50$	$-0.''11$	$-00.''01$
21.	Zlatokop	$+0.''17$	$+0.''45$	$+0.''06$	$+00.''78$	$-0.''25$	$-0.''12$	$-17.''20$
22.	Hisar	$-0.''27$	$+0.''20$	$-0.''02$	$+03.''61$	$-0.''07$	$+0.''13$	$-04.''76$
23.	Church in Trstenk	$-0.''30$	$+0.''14$	$+0.''06$	$+02.''40$	$-0.''42$	$+0.''11$	$+45.''11$
24.	Church in Čačak	$+0.''02$	$+0.''29$	$+0.''09$	$+02.''40$	$-0.''52$	$+0.''10$	$-02.''83$
25.	Starača	$+0.''16$	$+0.''26$	$-0.''11$	$+04.''61$	$-0.''32$	$+0.''24$	$-06.''29$
26.	Ozerovac	$+0.''90$	$+0.''09$	$-0.''09$	$+05.''60$	$-0.''08$	$+0.''25$	$-09.''38$
27.	Avala	$+0.''15$	$+0.''35$	$-0.''28$	$+01.''72$	$-0.''23$	$+0.''10$	$-00.''84$
28.	Kulič	$-0.''20$	$+0.''36$	$-0.''24$	$+00.''62$	$-0.''42$	$+0.''20$	$-09.''51$
29.	Podgorica	$+0.''00$	$+0.''10$	$-0.''18$	$+01.''92$	$-0.''30$	$+0.''10$	$-03.''54$
30.	Osojna	$-0.''20$	$+0.''35$	$-0.''15$	$+02.''10$	$-0.''30$	$-0.''38$	$+05.''50$

Due to the putting of the apparent star coordinates at our disposal the work is significantly simplified. The only thing to be done is to calculate the apparent right ascensions and declinations following the procedure described in Part IV. In this part of the work, as already said, one uses the baricentric coordinates of the Solar-System bodies calculated according to the JPL numerical theory DE200/LE200 for the purpose of obtaining the baricentric coordinates of the Earth and its velocity vector. The corresponding data concerning the interval 1899-1924 have been put at our disposal due to the courtesy of colleague B. Jovanović.

After obtaining the difference between the FK5 positions and the ones in Bošković's reduction sheets it becomes possible to reduce Bošković's results to the FK5 system. In these reductions one calculates the corrections of Bošković's results according to the differential formulae (Dačić 2000).

Table 6. contains the reductions obtained on the basis of the differences in the apparent places of FK5 and Bošković's ones calculated directly for every observation separately. In other words, after reducing the latitudes and azimuths to the FK5 sys-

Table 7. Plumb-Line Deflections reduced to Fk5 System ($[u]$ - corrected total value of plumb-line deflection; $[E]$ - corrected deflection angle; $[\lambda' - \lambda]$ - corrected difference between astronomical and geodetic longitudes; u, E and $\lambda' - \lambda$ - Bošković's numerical values).

	Place of Observation	$[u]$	u	$[E]$	E	$[\lambda' - \lambda]$	$\lambda' - \lambda$
1.	Paraćin	14″.07	14″.1	− 90°.61	− 90°.4	−19″.50	−19″.5
2.	Rtanj	9″.79	9″.1	− 44°.21	− 44°.6	− 9″.45	− 8″.8
3.	Midžor	8″.67	8″.6	− 11°.84	− 12°.1	− 2″.45	− 2″.5
4.	Trem	13″.26	13″.3	− 39°.31	− 38°.6	−11″.51	−11″.4
5.	Jastrebac	8″.81	8″.8	− 85°.64	− 82°.8	−12″.08	−11″.8
6.	Streser	12″.97	12″.3	− 63°.40	− 68°.6	−15″.77	−15″.7
7.	Petrova Gora	7″.00	6″.6	− 97°.63	− 96°.1	− 9″.49	− 9″.1
8.	Kopaonik	4″.00	3″.3	−134°.60	−136°.2	− 3″.91	− 3″.2
9.	Jankov Kamen	3″.47	3″.8	+ 46°.05	+ 54°.6	+ 3″.44	+ 4″.2
10.	Tornik	3″.39	3″.1	−160°.02	−170°.8	− 1″.61	− 0″.7
11.	Mali Povlen	7″.89	7″.6	− 68°.12	− 64°.4	−10″.20	− 9″.6
12.	Deli Jovan	6″.28	6″.1	+ 23°.75	+ 28°.2	+ 3″.53	+ 4″.0
13.	Veliki Sumorovac	4″.21	3″.9	− 80°.43	− 76°.7	− 5″.80	− 5″.3
14.	Crni Vrh	6″.50	6″.1	− 68°.61	− 68°.0	− 8″.38	− 7″.9
15.	Bukulja	8″.08	7″.5	− 48°.36	− 49°.3	− 8″.43	− 8″.0
16.	Cer	9″.33	7″.6	− 60°.24	− 64°.1	− 9″.97	− 9″.5
17.	Church in Niš	11″.62	10″.9	− 78°.63	− 81°.1	−15″.66	−14″.9
18.	Church in Zaječar	14″.97	15″.3	+ 87°.09	+ 88°.5	+20″.75	+21″.2
19.	Church in Negotin	15″.07	15″.2	+ 75°.60	+ 77°.4	+20″.37	+20″.6
20.	Pirot	2″.06	2″.4	−179°.72	+165°.4	− 0″.01	+ 0″.9
21.	Zlatokop	17″.22	16″.8	− 87°.40	− 89°.7	−23″.34	−22″.8
22.	Hisar	5″.97	6″.1	− 48°.99	− 52°.4	− 6″.53	− 6″.6
23.	Church in Trstenik	45″.17	45″.5	+ 86°.95	+ 86°.8	+62″.31	+62″.8
24.	Church in Čačak	3″.71	3″.1	− 49°.70	− 50°.2	− 3″.92	− 3″.3
25.	Starača	7″.80	7″.5	− 53°.76	− 55°.3	− 8″.81	− 8″.7
26.	Ozerovac	10″.92	10″.7	− 59°.16	− 63°.9	−13″.09	−13″.3
27.	Avala	1″.91	1″.7	− 26°.03	− 23°.6	− 1″.18	− 1″.0
28.	Kulič	9″.53	9″.3	− 86°.27	− 85°.7	−13″.39	−13″.1
29.	Podgorica	4″.03	3″.9	− 61°.53	− 58°.8	− 4″.98	− 4″.7
30.	Osojna	5″.89	6″.5	+ 69°.10	+ 71°.3	+ 7″.72	+ 8″.7

tem it becomes as if Bošković in his reductions had used the star positions from FK5, the new astronomical constants and the new procedure for the calculating of the apparent places.

The random error of results due to the one in star positions is equal to about $+0.''01$ for both latitude and azimuth. Even this error component in the mean latitude value can be even smaller depending on the number of observed Pevtsov's pairs at a given point (where the plumb-line deflection was determined).

The results of Bošković's astrogeodetic determinations are treated by reducing the coordinates of observed stars to the FK5 system. The latitude is calculated again and the results of latitude observations are analysed separately. After reducing to the CIO pole for every point the corrected total value of plumb-line deflection is calculated and the corrected deflection angle is obtained. On the basis of the comparative values from Table 7. it can be concluded that the introduced corrections have not changed Bošković's results significantly. It is of importance that these results are reduced to the modern FK5 reference system. In this way, by introducing corrections for the coordinates of the instantaneous pole a homogenisation in Bošković's results of latitude and azimuth determinations takes place.

4. CONCLUSION

A general conclusion from all presented above is the following:

- in the fundamental catalogues of the FK series there are enough data for the calculation of errors in the star positions reduced to a chosen epoch;

- the differential formulae enable to determine, in addition to the most favourable observational conditions, also the influences of the errors in the star coordinates on the results of geodetic astronomy;

- the corrections do not result in essential changes, but it is important that by introducing the corrected positions for the observed stars the results of Bošković's determinations of plumb-line deflections in Serbia are reduced to the reference system of FK5.

References

Bošković, S.P.: 1952, Skretanje vertikala u Srbiji, SAN, *Posebna izdanja*, **CXCVI**, Geografski inst., knj. 4.

Dačić, M.: 2000, Proceedings of the Second Serbian - Bulgarian Astronomical Meeting, June 23 - 26, 2000, Zaječar , Dimitrijević, M.S., Popović, L.Č. and Tsvetkov M. (eds.), *Publ. Astron. Obs. Belgrade*, **67**, 27.

Fricke, W., Kopff, A.: 1963, *Veröff. Astron. Rechen-Institut Heilderberg*, **10**.

Fricke, W., Schwan, H., Lederle, T.: 1988, *Veröff. Astron. Rechen-Institut Heilderberg*, **32**.

Fricke, W., Schwan, H., Corbin, T.: 1991, *Veröff. Astron. Rechen-Institut Heilderberg*, **33**.

Kopff, A.: 1937, Dritter Fundamentalkatalog des Berliner Astronom. Jahrbuchs. I Teil. Die Auwers-Sterne für die Epochen 1925 und 1950, *Veröff. Astron. Rechen-Institut Berlin-Dahlem*, **54**.

Mueller, I.I.: 1969, Spherical and Practical Astronomy as Applied to Geodesy, Frederick Ungar Publishing Co., New York.

Peters, J.: 1907, Neuer Fundamentalkatalog des Berliner Astronom. Jahrbuchs, *Veröff. Astron. Rechen-Institut*, **33**.

Зверев М.С.: 1950, Успехи астрономических наук, Том **V**, Изд. Акад. наук СССР, Москва – Ленинград.